COST BASED HETEROGENEOUS ACCESS MANAGEMENT IN MULTI-SERVICE, MULTI-SYSTEM SCENARIOS

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ABSTRACT

This paper covers the issue of how mobile users of different service classes should be assigned to a set of radio access technologies (RATs) with overlapping coverage from a practical perspective. The aim is to allocate users with minimum rate constraints to all RATs that the weighted sum of assignable users is maximized. We introduce the concept of resource costs to formulate an optimization problem, which is very expensive to solve (NP-complete). Using the approach of continuous relaxation, we develop a suboptimal radio access selection algorithm which is designed for practical applications and converges close to the optimum. Simulation results show considerable gains of the utility in comparison to a standard Load Balancing Strategy.

I INTRODUCTION

The emergence of new radio access technologies (RATs) and the interest of mobile operators to exploit their legacy systems often leads to multi-system scenarios. Today operators manage several air-interfaces with overlapping coverage and usually offer a set of basic services to their customers independent of the transmission technology. The down-compatibility of modern user equipment to legacy systems, primarily thought to allow a gradual development of infrastructure and to simplify maintenance, gives operators the freedom to assign users to an air-interface of their choice. This freedom proved to be beneficial in overload situations, and is reflected in the 3GPP release and in [1]. Here exchanged load information between RATs can be used to initiate vertical handovers of users to less loaded systems. Pure load balancing however, without consideration of the characteristics of users, services and air-interfaces is in general suboptimal. This was first discovered by Furuskär, who introduced a service based user assignment strategy in [2]. Based on the fact that the capability to support users differs between air-interfaces and depends on the service class, optimal service mixes for each RAT were derived. An algorithmic solution covering the issue of fairness is presented in [3] which is based on this concept. Considering only the service class, however, does in general not lead to an efficient use of system resources since important factors like the channel quality are neglected. This shortage is overcome in this paper. We present an user assignment strategy based on resource costs, that exploits what we call multi-system diversity.

We formulate an optimization problem that maximizes the weighted sum of assignable users based on their resource costs. The costs reflect how efficient an user can be supported by an air-interface and take the service class, the channel quality and characteristics of the air-interface into account. It is not considered to assign users partially to multiple air-interfaces due to their disjoint architecture. Under this assumption, the optimization problem is classified as non-polynomial (NP) complete and can in general not be solved in polynomial time. Therefore we present a suboptimal algorithm based on continuous relaxation. Performance bounds of such algorithms were presented in [4] in a more theoretic framework. Here we develop a suboptimal algorithm adapted especially to practical applications. In simulations we compare the performance of our algorithm with a basic Load Balancing Strategy in a heterogeneous UMTS- GSM multi-cell system and show considerable gains in terms of the utility.

II SYSTEM MODEL

Notations: In the following, small and large bold fonts denote vectors and matrices, respectively. Calligraphic letters are used for sets and $\lceil x \rceil$ ($\lfloor x \rfloor$) denote the closest larger (smaller) integer value to x.

We consider a scenario where a set of users $\mathcal{I} = \{1, \ldots, I\}$ is in the coverage area of a set of different radio access technologies $\mathcal{M} = \{1, \ldots, M\}$ and mobiles are able to support all technologies. Interference between different systems is precluded since each air-interface is assumed to use an individual non overlapping frequency band. Radio access technologies have in general only a limited amount of resources. In CDMA systems the maximum downlink transmission power is limited and distributed between the users, while in GSM the bandwidth is assigned in the form of orthogonal time-slots.

In our setting, each user requests a service, which corresponds to a guaranteed transmission rate. If user i is assigned to air-interface m, a certain percentage $c_{i,m},\ \ i, m \in \mathcal{I}, \mathcal{M}$ of the distributable resource of this RAT is needed to fulfill its service request. We refer to this share as the relative resource cost, which in general does not only depend on the service class and channel of the user but also on the characteristics of the air-interface. Dependence on the transmission technology can result from different granularities of distributable resources, modulation and coding schemes and robustness to interference.

In order to apply the cost concept efficiently we assume that the resource costs are independent of the compilation of assigned users. In this case we can assume that in a fully loaded

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system the resource costs are constant. Interference tolerating single-cell single-input/single-output scenarios with e.g. a sum power constraint and a SIR of the following form apply to our model:

$$SIR_{i,m} = \frac{h_{i,m}P_{i,m}}{\sum_{j \neq i} h_{i,m}P_{j,m} + \sigma^2} = \frac{P_{i,m}}{P_{sum,m} - P_{i,m} + \sigma^2/h_{i,m}} \quad (1)$$

Here $h_{i,m}$ denotes the channel gain, $P_{i,m}$ the power of user i in air-interface m and $\sum_{i \in \mathcal{I}} P_{i,m} = P_{sum,m} \leq P_{max,m}$. For a given target SIR $\gamma_{i,m}$, which corresponds to the minimum requested rate r_i of user i in RAT m, we can give an expression for the resource costs by using (1) and $P_{sum,m} = P_{max,m}$:

$$c_{i,m} = \frac{P_{i,m}}{P_{max,m}} = \frac{1 + \sigma^2/(h_{i,m}P_{max,m})}{1/\gamma_{i,m} + 1} \quad (2)$$

For this class of systems, $c_{i,m}$ represents the needed percentile of available transmission power under the assumption of full load. To extend the model to multi-cell scenarios σ^2 represents the variance of the the inter-cell interference and the thermal noise. Using this simplification we neglect that changing the power in one cell influences the interference of neighbors and therefore can affect the own resource assignment. However, this effect is small, since we are interested in operation at the load limit with maximum transmission power.

In TDMA systems with a maximum number of time-slots S_{max} with fixed length and under the constraint that users cannot share time-slots the costs can be calculated by

$$c_{i,n} = \frac{1}{S_{max}} \lceil r_i / R(P_{max,n}h_{i,n}/\sigma^2) \rceil. \quad (3)$$

Here $R(\cdot)$ gives the rate corresponding to the target SIR. Using the costs of all users in all RAT, we define an $M \times I$ cost matrix $\mathbf{C}$ and $[\mathbf{C}]_{i,m} = c_{i,m}$. The association of each user with the set of air-interfaces is represented in the $M \times I$ assignment matrix $\mathbf{V}$, with elements

$$v_{i,m} = \begin{cases} 1 \text{ if user } i \text{ is assigned to air-interface } m \\ 0 \text{ else.} \end{cases} \quad (4)$$

III Optimization Problem and Relaxation

In our setup, the optimization problem is formulated from an operator's perspective with the aim to maximize a utility function, represented by the weighted sum of assignable users. In the case that all weights are equal, this relates to allocate as many users as possible, otherwise it allows priorization of certain user or services classes. From a cross-layer perspective the weights could also represent the coupling between physical and higher layers. To cope with the cost principle we limit our optimization model to a single-cell system with M basestations, one corresponding to each RAT. Multi-cell scenarios are covered by this concept by formulating one individual, decoupled optimization problem for each cell under the assumption of constant inter-cell interference. In multi-system scenarios, it is in general not an option to split service requests and assign users to multiple air-interfaces at the same time. Under these premises the optimization problem can be written as follows:

$$\begin{aligned} y_{opt} = \max_{\mathbf{V}} & \sum_{i,m \in \mathcal{I},\mathcal{M}} w_i v_{i,m} \\ \text{subj. to} & \sum_{i \in \mathcal{I}} v_{i,m} c_{i,m} \leq 1 \ \forall m \in \mathcal{M} \\ & \sum_{m \in \mathcal{M}} v_{i,m} \leq 1 \ \forall i \in \mathcal{I} \\ & v_{i,m} \in \{0,1\} \ \forall i,m \in \mathcal{I},\mathcal{M} \end{aligned} \quad (5)$$

Here w_i denotes the weight of user i. The first set of constraints in (5) assures that no more resources than available are assigned to each air-interface, the second one prevents multiple assignment of each user. In order to avoid splitting or partial assignment of users, the third constraint is used, which however leads to the combinatorial nature of the problem with exponentially growing complexity in the degrees of freedom. Problem (5) can be identified as the Generalized Assignment Problem (GAP), which is NP-complete [5]. Thus, for practical assignment strategies, we are restricted to use suboptimum algorithms in most cases.

We can transform problem (5) into a convex problem if we relax the third constraint and allow fractional assignment of users. We then obtain the following representation:

$$\begin{aligned} y^* = \max_{\mathbf{V}} & \sum_{i,m \in \mathcal{I},\mathcal{M}} w_i v_{i,m} \\ \text{subj. to} & \sum_{i \in \mathcal{I}} v_{i,m} c_{i,m} \leq 1 \ \forall m \in \mathcal{M} \\ & \sum_{m \in \mathcal{M}} v_{i,m} \leq 1 \ \forall i \in \mathcal{I} \\ & v_{i,m} \geq 0 \ \forall i,m \in \mathcal{I},\mathcal{M}, \end{aligned} \quad (6)$$

where $\mathbf{V}^*$ is the optimum relaxed assignment. Based on this relaxed optimization problem we will design our assignment strategy as presented in section IV.

IV Algorithm

In this paper we present an assignment strategy designed for practical applications. We therefore have to take user mobility, varying channel-gains and stochastic service requests into account, which make the costs and requests time varying. To avoid a dynamic problem formulation we split the design of our assignment strategy into two parts: In the first one, we derive an algorithm to solve (5) under the assumption of constant cost-values, which we refer to as "snapshot optimization". In the second part we suggest triggers when and in which cells the snapshot optimization should be initiated. It seems to be inevitable to find efficient triggers since the system performance gain needs to be related to the expenses for each snapshot optimization in terms of signaling cost and in terms of computational effort. Although tight performance bounds can be derived for the snapshot optimization, finding optimum triggers for its execution analytically is difficult. Thus, we evaluate these triggers by simulating strategies with different tigger types in section V.

A Snapshot optimization

Based on the solution of (6), which represents a Linear Program, a feasible solution to the original problem can be deduced by assigning $\lfloor V^* \rfloor$. It was shown in [4], that using this procedure there are at most M users less assigned than in the optimum solution of (5). There exists a variety of algorithms in the literature for solving Linear Programs. Popular among them are the simplex, the cutting plane and the ellipsoid method [6], [7]. We will use a simplex approach as basis for our algorithm, although convergence to the optimum in polynomial time is not proven. This choice is motivated by the advantage of this simplex approach, that the air-interface weights of the preceding snapshot optimization can be used as a starting point in algorithm 1. Since changes of the cost-matrices are limited convergence of the procedure is usually achieved in few steps. The snapshot optimization can be solved using the following concept, which is first presented informally to improve readability:

1. An optimum air-interface weight λ'_m is evaluated for each RAT (using the simplex method).
2. Based on the weights, each user i is then assigned to the candidate list of a RAT $m_{i,sl} = \arg\min_m \lambda'_m c_{i,m}$. Thereby the original problem is decoupled into M optimization problems.
3. Each RAT can now maximize the weighted number of assignable users based on its candidate list, which is independent of the other RATs.

Although the procedure above seems pretty simple we will show that it gives a tight approximation to (5). The weight interpretation is based on the Lagrangian theory: By incorporating all constraints, we can build the Lagrange function of (6)

$$L(\mathbf{V},\boldsymbol{\lambda}) = \sum_{i,m\in\mathcal{I},\mathcal{M}} w_i v_{i,m} - \sum_{m\in\mathcal{M}} \lambda'_m (\sum_{i\in\mathcal{I}} c_{i,m} v_{i,m} - 1) - \sum_{i\in\mathcal{I}} \lambda''_i (\sum_{m\in\mathcal{M}} v_{i,m} - 1) + \sum_{i,m\in\mathcal{I},\mathcal{M}} \lambda'''_{i,m} v_{i,m}. \quad (7)$$

Here, λ are the dual parameters, which have to be non-negative. Since the problem is convex, we know that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient at the optimum solution. That is, the derivation of the Lagrangian function after the primal variables $v_{i,m}$ has to be zero as well as for all constraint in connection with the dual variables holds:

$$\frac{\partial L(\mathbf{V},\boldsymbol{\lambda})}{\partial v_{i,m}} = w_i - \lambda'_m c_{i,m} - \lambda''_i + \lambda'''_{i,m} = 0 \quad \forall i,m \in \mathcal{I},\mathcal{M} \quad (8)$$

$$\lambda'_m (\sum_{i\in\mathcal{I}} c_{i,m} v_{i,m} - 1) = 0 \ \forall m \in \mathcal{M} \quad (9)$$

$$\lambda''_i (\sum_{m\in\mathcal{M}} v_{i,m} - 1) = 0 \ \forall i \in \mathcal{I} \quad (10)$$

$$\lambda'''_{i,m} v_{i,m} = 0 \ \forall i,m \in \mathcal{I},\mathcal{M} \quad (11)$$

Given the optimum $\lambda'_m \ \forall m \in \mathcal{M}$ we can deduce from (8) the optimum air-interface for each assigned user. Without loss of generalization assume a scenario with two RAT $m,n \in \mathcal{M}$ and $\lambda'_m c_{i,m} < \lambda'_n c_{i,n}$ for user i. Then for both RAT (8) has to hold in the optimum:

$$w_i - \lambda''_i = c_{i,m}\lambda'_m - \lambda'''_{i,m} = c_{i,n}\lambda'_n - \lambda'''_{i,n}. \quad (12)$$

If user i is assigned, then either $\lambda'''_{i,m}$ or $\lambda'''_{i,n}$ has to be zero because of (11) and it follows due to positivity of the dual parameters that user i can only be assigned to RAT m. Therefore, λ' balances the user assignments and can be interpreted as a measure of the air-interface capacity. In case of high average costs in one RAT, a low λ' ensures that users are assigned to this air-interface. Although many algorithms exist to solve Linear Programs the usage of the dual variables as air-interface weights allows a simple, dezentralized evaluation of the assignment and gives some insights into the problem structure. Generalizing this concept for given air-interface weights, all users can be split into the subsets

$$\mathcal{I}_m = \{i | i \in \mathcal{I}, m = \arg\min_{m\in\mathcal{M}} c_{i,m}\lambda'_m\} \quad (13)$$

which represent the candidate lists of all RATs and decouple the optimization problem. In case of $\lambda'_m c_{i,m} = \lambda'_n c_{i,n}$ user i could be member of multiple candidate lists $i \in \mathcal{I}_m, \mathcal{I}_n$, which would not allow a complete decoupling. This events however are seldom and decoupling can be achived by chosing one candidate list randomly for the user.
Based on the candidate lists, we then obtain M individual optimization problems

$$\begin{aligned} y_m(\boldsymbol{\lambda'}) = & \max_{\mathbf{V}} \sum_{i\in\mathcal{I}_m(\boldsymbol{\lambda'})} w_i v_{i,m} \\ & \text{subj. to} \sum_{i\in\mathcal{I}_m(\boldsymbol{\lambda'})} v_{i,m} c_{i,m} \le 1 \\ & v_{i,m} \in \{0,1\} \ \forall i \in \mathcal{I}_m(\boldsymbol{\lambda'}) \end{aligned} \quad (14)$$

which give the utility of the original problem by

$$y(\boldsymbol{\lambda'}) = \sum_{m\in\mathcal{M}} y_m(\boldsymbol{\lambda'}). \quad (15)$$

Equation (14) represents the well known Knapsack problem, which is also NP-complete [8]. We can solve it exactly for equal weights and approximate it fairly good for arbitrary weights using the following procedure: We assign users one by one, always choosing the user with the greatest $w_i/c_{i,m}$ ratio in $\mathcal{I}_m$ that is not already assigned, as long as the sum costs in this RAT are not greater than one [8]. Using the described assignment policy, we will obtain a feasible solution to (5) close to the optimum, if the optimum $\boldsymbol{\lambda'}$ is known.
To obtain the optimum $\boldsymbol{\lambda'}$ we use an iterative simplex like approach. We evaluate (15) by starting with an arbitrary $\boldsymbol{\lambda'}$. Then, we determine a gradient by decreasing all λ'_m one by one until (15) changes. Those changed air-interface weights, that maximize $y(\boldsymbol{\lambda'})$, build the weights for the next iteration. This procedure is repeated until (15) does not increase anymore. Due

Algorithm 1 Simplex Algorithm

(1) Initialize $\boldsymbol{\lambda}'^{(0)}$ arbitrary
while $y(\boldsymbol{\lambda}'^{(k)}) > y(\boldsymbol{\lambda}'^{(k-1)})$ or $k < 2$ **do**
  (2) calculate y from (15)
  for $m = 1$ to M **do**
    (3) determine weights $\tilde{\boldsymbol{\lambda}}^m$ corresponding to neighboring vertices of the simplex $[\tilde{\boldsymbol{\lambda}}^m]_n = \lambda_n'^{(k)}\ \forall n \neq m$ and

$$[\tilde{\boldsymbol{\lambda}}^m]_m = \max_{n \neq m, i \notin \mathcal{I}_m} \frac{c_{i,n}}{c_{i,m}} \lambda_n'^{(k)}$$

  end for
  (4) set $\boldsymbol{\lambda}'^{(k+1)} = \arg\max_m y(\tilde{\boldsymbol{\lambda}}^m)$
end while
(5) return $\boldsymbol{\lambda}'^{(k-1)}$ and corresponding assignment

to the convexity of the problem, the corresponding air-interface weights are the optimal ones (see Algorithm 1).

B Triggers for Snapshot Optimization

We now present two cost based strategies which use different events to trigger a snapshot optimization. Both are adapted to a multi-cell scenario where each trigger initiates the optimization only for one superimposed cell at a time; in the simulation scenario a superimposed cell corresponds to the coverage area of one colocated GSM and UMTS basestation. When the snapshot optimization procedure is triggered the costs are evaluated for each existing call inside the superimposed cell for each RAT. According to Algorithm 1 then the centralized Multi Radio Resource Management (MRRM) calculates the optimum air-interface weights and assignment. Alternatively to signaling the resulting assignment matrix, the assignment can also be obtained decentralized in the individual RATs based on the air-interface weights. As a result, each user is either kept in its current system, vertically handovered or dropped from the system.

In the Cost Based Strategy 1, the snapshot optimization is triggered before each call setup for that superimposed cell which is closest to the new user. It is executed without considering the new user, who may be blocked thereafter if no further assignment is possible. This strategy is characterized by that the optimization does not drop any ongoing call, since a feasible assignment already existed before its execution.

The Cost Based Strategy 2 triggers the snapshot optimization whenever an existing call can no longer be supported within the current RAT; the optimization is then performed in that superimposed cell to which that call is connected to. The performance of both cost based strategies is compared to the MRRM strategy Load Balancing and to Separated System operation.

The Load Based Strategy provides the same general MRRM functionality, which is also present in both cost based strategies, however no snapshot optimizations are triggered.

At call setup, the new call is attempted to be assigned to its default RAT. In case not enough resources are available there the alternative RAT is considered, also incorporating all neighboring cells. If none of the RATs can support the new user it is blocked. In case an ongoing call cannot be supported in a RAT anymore an inter-system handover (IS-HO) to the alternative RAT is tried. As at call setup also the neighboring cells are considered. If the alternative RAT also cannot support the call because of high load the call is dropped from the system. The Separated System strategy is taken as a reference, where no IS-HOs are possible. Each user is only served in its default RAT, and blocked or dropped immediately in the case of resource shortage.

V Simulation Results

A dynamic event-driven system level simulator [1], [9] has been implemented in C++ based on the IKR Simlib. As described there in detail, it consists of a heterogeneous multi-cell environment with colocated UMTS and GSM cells. In the simulation scenarios, each GSM basestation has 21 time slots, in UMTS the transmission power has a maximum value of 16,6 W per cell. Fading is switched off in order to emphasize that the gain of the cost based strategies does not rely on those channel variations. All users have exactly one link per connection, i.e. only hard handovers are allowed. The simulations were carried out with pure real time service with a guaranteed datarate of 12,2 kbit/s and an average call duration of 120 seconds, such as voice telephony. The amount offered traffic was varied up to very high traffic loads. In oder to measure the maximum reachable cell throughput, no cell resources are reserved at call setup, i.e. all new calls are accepted as long as any cell resources are available. All users move with 120 km/h according to a vehicular mobility model. Each RAT has its own RRM which is in charge of controlling intra-system handovers, and there is a centralized MRRM entity which controls IS-HOs according to its MRRM strategy.

Figure 1 shows the average number of assigned users per cell, which represents the utility with equal weights, for Separated System operation and for the three MRRM strategies Load Balancing, Cost Based Strategy 1 and 2. Since no resources are reserved for handovers at call setup, dropping exceeds blocking by approximately one order of magnitude at vehicular mobility and only the sum of both is presented in Figure 2. A service denial quota of 2.5% could be a reasonable system configuration point for operators. At this point, the Cost Based Strategies 1 and 2 show a gain of 15-20 % of the utility, represented by the amount of users in the system, in comparison to the MRRM strategy Load Balancing. The performance gain of the cost based strategies arises from frequent user re-assignment to their optimal air-interface and relies on the channel and service dependent suitability of the UMTS and GSM air-interface. As the GSM resource costs, based on time slots, are basically distance independent, while the required UMTS resource costs in terms of transmission power increases strongly with the distance, it is observed that the cost based strategies minimize the resource consumption by increasing the UMTS user density around the cell center, while distant users are preferably assigned to GSM. At vehicular mobility the differences between both cost based strategies

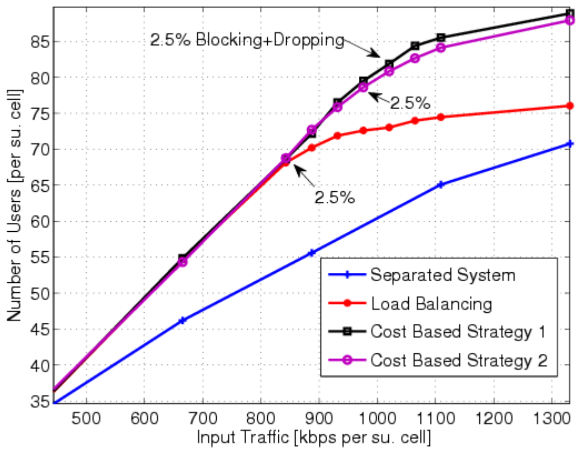

Figure 1: Utility of assignment strategies

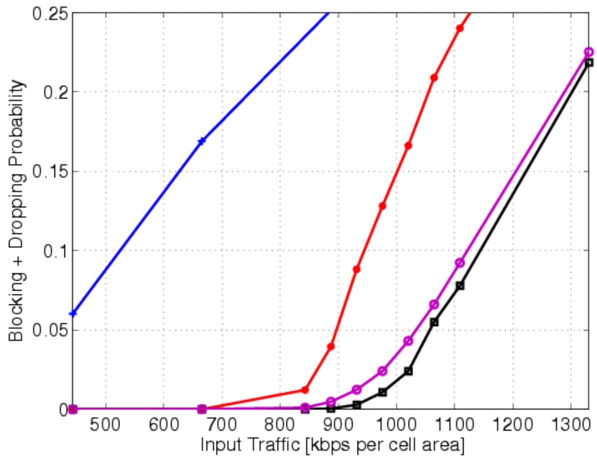

Figure 2: Service denial probability

are not too large since the snapshot procedure is called sufficiently often with 0.7 and 0.4 calls per second per cell. At lower mobile speeds the triggers of Strategy 2 occur less frequently which leads to larger performance gaps between both cost based strategies. To get a fair comparison between the cost based strategies and Load Balancing one has to compare the signaling and computational efforts. Strategy 1 and 2 need approximately 6 iterations per optimization call to calculate the optimum air-interface weights and initiate 3 IS-HO per superimposed cell per second compared to 0.3 IS-HO when applying the Load Balancing Strategy. Although these expenses seem to be high for practical scenarios and costs for handovers and signaling as well as signaling delays are neglected, the results can be used as benchmark to evaluate the performance of further simplified MRRM strategies.

VI Conclusion

A cost based heterogeneous access selection strategy was derived for multi-system scenarios. We introduced the concept of resource-costs, where the channel gain, service class and characteristics of the radio access technology are bundled into one cost parameter per user for each RAT. Based on this cost values, we derived an algorithm that calculates an assignment close to the maximum weighted sum of assignable users. Due to the fast convergence, 3-6 iterations in average, and the simple, decentralized RAT selection procedure for each user based on air-interface weights, the algorithm seems to be promising for practical applications.

The performance of the variants of the cost based algorithm was evaluated in a heterogeneous GSM-UMTS multi-cell simulation environment. At high load, the amount of carried traffic increased by an amount in the order of 10-20% in comparison to the Load Balancing Strategy. This gain arises from the more efficient exploitation of resources. Although, the gains come at the cost of an impractically high signaling effort and amount of inter-system handovers, the results can serve as valuable benchmark for further simplified algorithms.

References

[1] M. Hildebrand, K. David, G. Piao, R. Sigle, D. Zeller, and I. Karla, "Performance Investigation on Multi Standard Radio Resource Management," in *IST Summit on Mobile & Wireless Communications*, 2004.

[2] Anders Furuskär and Jens Zander, "Multiservice Allocation for Multiaccess Wireless Systems," *IEEE Transactions on Wireless Communications*, vol. 4, no. 1, January 2005.

[3] Ingmar Blau and Gerhard Wunder, "Optimal service allocation in multi-system scenarios with linear subsystem capacity regions," in *The 9th Symposium on Wireless Personal Multimedia Communications (WPMC '06)*, San Diego, USA, 17th-20th September 2006.

[4] Ingmar Blau and Gerhard Wunder, "User allocation in multi-system, multi-service scenarios: Upper and lower performance bound of polynomial time assignment algorithms," in *41st Conference on Information Sciences and Systems (CISS '07)*, Baltimore, USA, March 2007.

[5] L. Fleischer, M.X. Goemans, V.S. Mirrokni, and M. Sviridenko, "Tight Approximation Algorithms for Maximum General Assignment Problems," in *Proceedings of the 17th ACM-SIAM Symposium on Discrete Algorithms*, 2006.

[6] Christos H. Papadimitriou and Kenneth Steiglitz, *Combinatorial Optimization Algorithms and Complexity*, Prentice-Hall Inc., 1982.

[7] R. M Freund and C. Roos, "The Ellipsoid Method," www.isa.ewi.tudelft.nl/∼roos/courses/wi485/ellips.pdf.

[8] Hans Kellerer, Ulrich Pferschy, and David Pisinger, *Knapsack Problems*, Springer, Berlin, Germany, 2004.

[9] Guihua Piao, Klaus David, Ingo Karla, and Rolf Sigle, "Performance of Distributed MxRRM," in *Personal, Indoor and Mobile Radio Communications (PIMRC'06)*, 2006.